TRANSFORMATIONS AND ANIMATIONS IN POLAR GRAPHING

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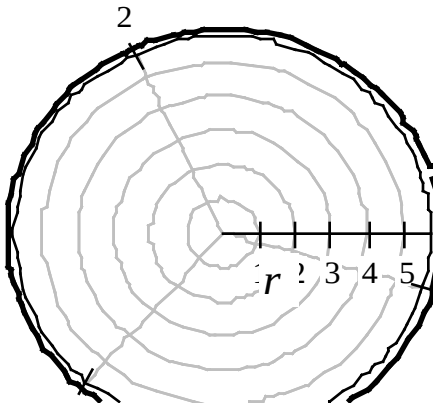
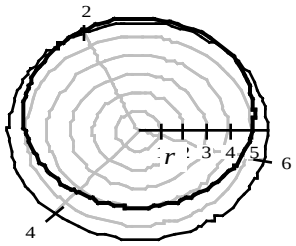
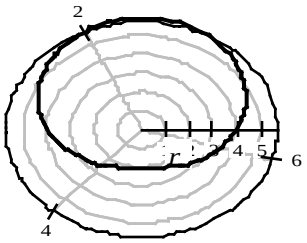
The polar graph of: **r = a + b sin (t) ,** for various values of **a** and **b,** is called a **Limaçon.** It can be better understood as a transformation of one circle, **r = 6**, into another, **r = 6 sin (t)**, as the relative values of **a** and **b** change. At one extreme: a = 6 and b = 0, and at the other extreme: a = 0 and b = 6. Using a powerful graphing utility such as *THEORIST* or *MATHEMATICA*, you can also set up an animation that will allow the transformation to take place before your eyes.



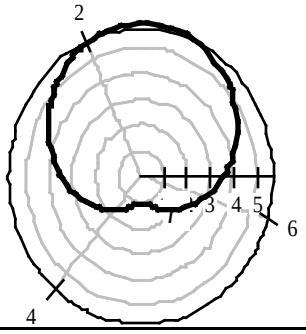
a = 6: r = 6 + 0·sin (t)



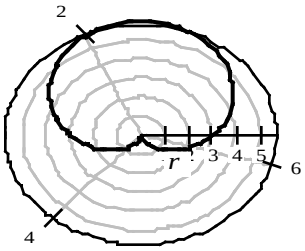
a = 5 r = 5 + 1·sin (t)



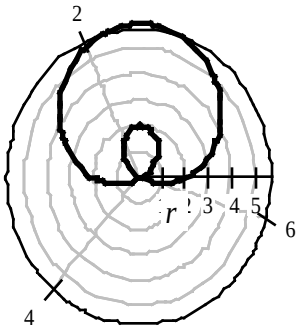
a = 4 r = 4 + 2·sin (t)



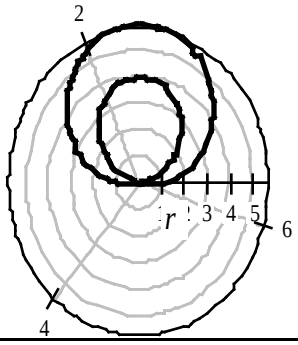
a = 3.5 r = 3.5 + 2.5·sin (t)



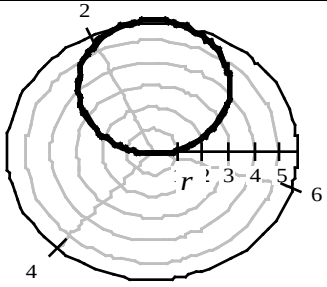
a = 3 r = 3 + 3·sin (t)



a = 2 r = 2 + 4·sin (t)



a = 1 r = 1 + 5·sin (t)



a = 0 r = 0 + 6·sin (t)

The circle, **r = 6** (A degenerate case, where **b = 0.**) is transformed into a limaçon with **a > b**. This transforms into the special case of a cardioid: **r = 3 + 3 sin(t)**, where **a = b**. When **a < b**, we have a double loop limaçon. Finally, transformed to the other degenerate case, a circle: **r = 6 sin (t),** where **a = 0.**