Mathematical Investigations
Trigonometry - Beyond the Right Triangles
Law of Cosines

Purposes:

To introduce and prove the Law of Cosines.
This activity sheet will lead students through a derivation of the Law of Cosines, first for a specific case and then in the general case. Several basic exercises follow the derivations.

Prerequisites:

(1) Students should be familiar with the standard labeling of triangles and triangle trigonometry.
(2) Students should be familiar with the Law of Sines.

Notes:

This sheet follows nicely from the sheet on the Law of Sines, though it is probably appropriate to include more practice problems before proceeding to this worksheet. Though the directions throughout the activity sheet are fairly specific from line to line, students will definitely need some direct teacher assistance. Still, we believe it is beneficial for the students to try to work through these derivations in groups rather than having the teacher lead them through all of the work.

It is recommended that more practice be given after these derivations which include open problems for both the Law of Sines and the Law of Cosines.

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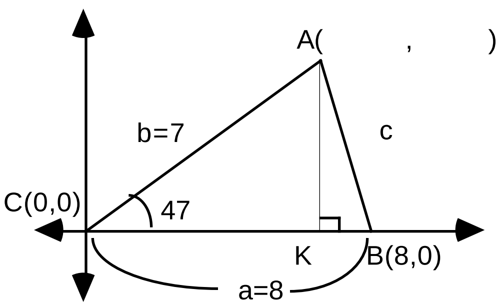
Name: ______________________

Solving a Triangle (second view)



Given ΔABC with side b=7 cm, side a=8 cm and angle C = 47°, find side c.

Try the Law of Sines:

Any problem?

Using the angle C = 47° and the side "b=7", and right-triangle trigonometry, find the coordinates of A(CK, KA). Leave answers in exact form e.g. 7cos47 instead of 4.774

A = (,)

Now, what are the coordinates of K? K = (,)

What's the length of KB? KB =

It is now possible to use the Pythagorean theorem on ΔAKB, to find c. Do the algebra below, but save the "arithmetic" (actual calculations) until the very end.

$c^2 = AK^2 + KB^2 = (\quad)^2 + (\quad)^2$ substitute

$c^2 =$ mult.out and collect terms

hint? $49(\sin(47°))^2 + 49(\cos(47°))^2 = ?$

$c^2 =$ with nothing "calculated"

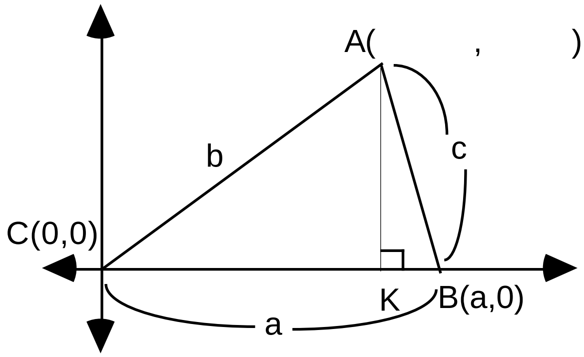
$c^2 =$ or $c =$ (now calculate)

Now finish "solving the triangle" by finding the angles: $A =$

$B =$

On the next page, we will generalize the preceding idea by following a similar procedure using all variables rather than specific values.

The Law of Cosines:



Find the coordinates of points A and K in terms of side b and ACB.

A =

K =

Find the length of KB.

Use the Pythagorean theorem on AB.

$c^2 =$

Multiply out and simplify, using a trigonometric identity.

$c^2 =$

What you have developed is called the Law of Cosines.

The Law of Cosines

In any triangle,

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

This might also be called the "Super Pythagorean Theorem" (SPT) Why?

Consider what would happen if we were to relabel the figure at the top of this page, replacing A with B, B with C, and C with A, and then reprove the Law of Cosines. We could derive the law for a^2 instead of c^2. Doing this would yield:

$a^2 =$

Similarly, we could relabel and derive the law for b^2. If we do, this would yield:

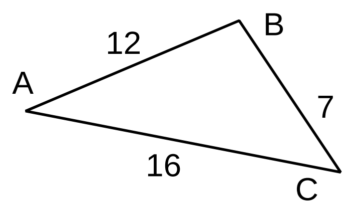
$b^2 =$

What happens to the formula above if the included angle is 90^o?

Now it's your turn.

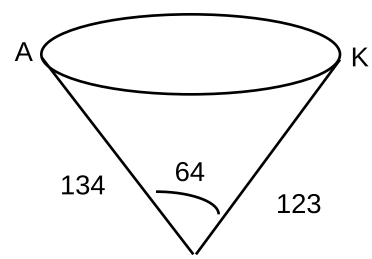
1. ΔXYZ, $x = 6$, $y = 8$, and $Z = 150°$. Find the <u>exact</u> value of z.

2. Find B



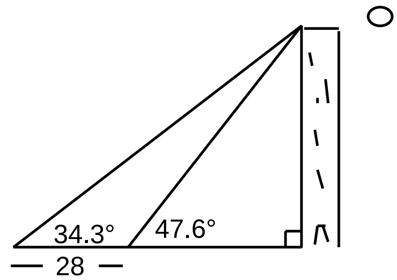
3. ΔJKM, $j = 19$, $k = 14$, and $m = 17$. Find K.

4. A surveyor needed to find the distance across an elliptical pond, labeled AK in the diagram. From one point on the side of the pond, the distances of 123 ft and 134 ft were found and the angle measured 64 degrees. What is the required distance?

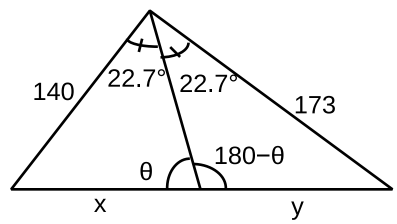


5. Given lengths of 15, 9, and 25, use the Law of Cosines to find one angle. Explain any "difficulties" encountered.

6. A spotlight on top of Jordan Towers makes an ellipse with a major axis of 28 feet on the ground. The angles to the top of the building from the top and bottom of the ellipse, were measured using a transit. Find the height of the building.



7. Find (x,y).



9. a. $\sin\left(\tan^{-1}\left(\frac{2}{3}\right)\right) =$

b. Solve: $\tan(3x+4) = \frac{2}{3}$ for $0 < x < 90^{o}$

10. a. Given $\tan(x) = \frac{2}{3}$, find sin(x).

b. Solve: $\tan(3x+4) = -\frac{2}{3}$ for $0 < x < 180^{o}$