IMSA

Mathematical Investigations
The Logarithm - Definition

Purpose:

To give a first glance at logarithms and logarithmic functions.

Prerequisites:

(1) Students are expected to be familiar with the graphs of exponential functions.
(2) Students are expected to have some understanding of inverse functions. This includes the graphical relationship where a function f and its inverse f^{-1} are symmetric with respect to the line $y = x$. Also, if the graph of f contains the point (a, b), then the graph of f^{-1} will contain the point (b, a).
(3) Students should be familiar with the concept of the domain and range of a function and be able to find them for reasonable functions.
(4) Students should be somewhat familiar with the number e. That is, they should know it is a special constant where $e \approx 2.718$. Note the quick mention of "ln," the natural logarithm, in problem 3.

Notes:

As stated, this activity sheet really is designed to give only the first introduction to logarithms. We believe it is important for students to see logs initially as inverse functions to help connect their knowledge of functions and exponents in order to help with logs as soon as possible. This needs to be followed by much more practice with graphs, switching forms of an equation, evaluation, and properties of logarithms.

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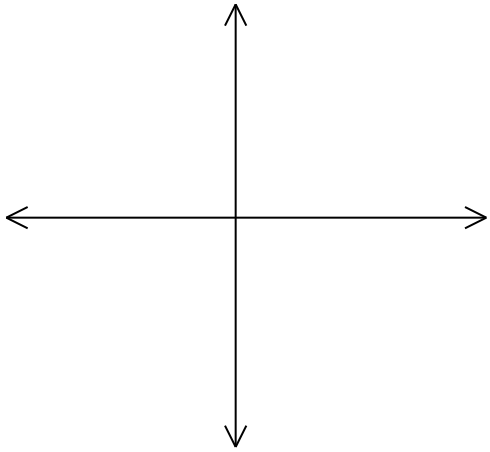
Name:_____

1. Consider the exponential function $f(x) = 10^x$.

   a. Graph the function and label 3 points.

   b. Graph $y = f^{-1}(x)$, the inverse function, and label three points.

   c. Write a sentence or two describing f^{-1}. Is f^{-1} a function? What is its domain? range?



Let's call f^{-1} the **logarithmic function** of x with a base of 10. Since this is obviously too long to write down each time, let's use the following abbreviation:

$$f^{-1}(x) = \log_{10}(x) \text{ or } y = \log_{10}(x)$$

2. On your calculator, graph both the functions $y = 10^x$ and $y = \log(x)$.
   [Note: On your calculator, $\log(x) = \log_{10}(x)$]

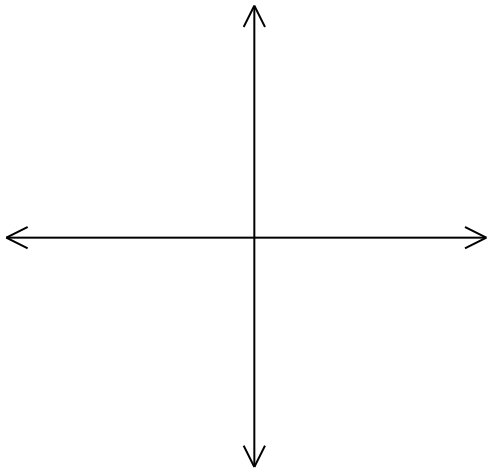
   The graphs of $y = 10^x$ and $y = \log_{10}(x)$ are inverse functions. This means that if $v = 10^w$, then $w = \log_{10}(v)$. Note that since the logarithm of v equals w, and w is an exponent; then the logarithm is an exponent.

3. Consider the exponential function $f(x) = e^x$.

a. Graph the function and label 3 points.

b. Let $g(x) = \ln(x) = f^{-1}(x)$, the inverse function. Graph $y = g(x)$ and label three points.

c. Write a sentence or two describing f^{-1}.
Is f^{-1} a function?
What is its domain? range?



Definition of a Logarithm:

$\log_b a = c$ if and only if $b^c = a$, $a > 0, b > 0, b \neq 1$

OR

$\log_{base}$ (number) = exponent if and only if $(base)^{exponent}$ = number, number > 0, base > 0, base ≠ 1

Examples: $\log_5 (25) = 2$ because $5^2 = 25$

$\log_2(256) = 8$ because $2^8 = 256$

Since $9^{3/2} = 27$, $\log_9(27) = 1.5$

Since $343^{1/3} = 7$, $\log_{343}(7) = 1/3$

Rewrite the following logarithmic equations as exponential equations and write the exponential equations as logarithmic equations.

$\log_3(81) = 4$ _____ $7^3 = 343$ _____

$\log_4(0.25) = -1$ _____ $\log_x(y) = z$ _____

$36^{(1/2)} = 6$ _____ $e^a = b$ _____