Number Theory Course Description
Fall 2017

Course Description:

Number Theory re-acquaints students to the number system they have been familiar with since they first learned 1, 2, 3. Students learn how the number system really fits together and how to exploit this structure to discover new relationships among the numbers and new applications for these relationships. The course serves as a stepping stone to higher mathematics by introducing students to rigorous proof and deep abstraction while remaining in mostly familiar territory. Curious students who want to get a glimpse of the powerful techniques and get a taste of higher mathematics but who are unsure of their readiness to handle the full abstraction of the more difficult pieces of material may elect a pass/fail option.
This course is *not* intended to be a college level course, but more of a survey course to introduce students to what Number Theory has to offer.

Instructor:
- Name: Carlo Ordoñez
- Office: A157 (Math office)
- Telephone number: x5477
- Email address: cordonez@imsa.edu

Office Hours:
- A days – Mods 2 and 7
- B days – Mod 5, after school until 5
- C days – Mod 6, after school until 5
- D days – Mod 2
- I days – 1-2 PM

Course Information
Room: A148
Section 1: A, B, C, D-days, Mod 3
Section 2: A, B, C, D-days, Mod 8

Text(s) / Materials:

We will not formally use a text this semester. There are many excellent textbooks that a student may use for reference or for additional practice problems, available in IMSA's IRC and online. The primary source material for the course is the lectures and discussions which occur in the classroom, and students will be expected to keep careful notes.

I may use *Number Theory* by Andrews and *A Friendly Introduction to Number Theory* by Silverman for guidance and additional exercises, but you do not need to purchase this. The former book is a Dover publication, so you can purchase that relatively cheaply if you'd like.

Additional reading materials may be distributed by the teacher to emphasize particular topics.

Grading Policies

Homework packets will be distributed roughly every week. They are due at the beginning of class; anything turned in after that time will be counted as late. For each day an assignment is late, 20% of earned points will be deducted from the grade.

Quizzes will be given roughly every couple of weeks going over the homework distributed the previous two weeks. They may be open note, where you may use any handouts or notes taken in class. For those taking notes by computer, you may use a printed copy of your notes.

Starting the second quarter, we will devote time in class to a final paper and presentation. Students may work with a partner and pick a topic related to number theory (which must be approved) to produce a 15-20 page research paper and a 20-25 minute class presentation by the end of the semester. Details will be distributed at the beginning of second quarter.

Grade scale: A: 89% or above, A-: 85-89%, B+: 81-85%, B: 74-81%, B-: 70-74%, C+: 66-70%, C: 59-66%, C-: 55-59%, D: < 55%

The grade will be broken down as follows:

Quarter: 60% quizzes, 40% homework

Semester: 48% quizzes, 32% homework, 20% final project

Student Expectations:

Students in Number Theory are expected to:

- Complete a large number of homework assignments in a timely manner. Homework is assigned to help students learn material in time for it to be applicable to the next section of material. Thus late homework exacts an intellectual penalty in not being prepared. It is also given a grade penalty to reflect its lateness.
- Keep current with course material. This material builds on itself, and not having complete understanding of today's material will make it much more difficult to learn tomorrow's.
- Keep good course notes.
- Practice with problems beyond the assigned homework, if they expect to earn high grades.
- Participate in class discussions.
- Seek help early if concepts are not clear.

Sequence of Topics and Activities

- Representation systems
    - Ways to represent numbers, base *b* representations
    - Division algorithm
    - Other positional systems
    - Zeckendorf's Theorem
- Special properties of the integers
    - Well-ordering
    - Induction
- Divisibility, factorization, and primes
    - Primes and factoring
    - GCD and Euclidean algorithm
    - Fundamental Theorem of Arithmetic
    - Linear Diophantine equations
- Equivalence relations
    - Relations
    - Defining new operations
- Modular arithmetic
    - Mechanics of modular arithmetic
    - Linear congruences
    - Chinese Remainder Theorem
- Euler's totient function and reduced residue systems
    - Reduced residue systems
    - Euler's function and its properties
    - Theorems of Fermat, Euler, and Wilson
    - Introduction to pseudoprimes
- Order and primitive roots
    - Powers and polynomials in modular arithmetic
    - Roots of unity
    - Order and primitive roots
- Cryptography
- Rational and real numbers
    - Constructing the rational and real numbers, and proving their properties
    - Reinvestigating the nature of positional notation systems, especially as applied to rational numbers
    - Cardinality
- Axiomatic treatment of the integers/natural numbers
    - Axioms for equals, addition, multiplication, and ordering
    - Looking at basic principles that aren't as basic as you thought they were!
    - Defining multiplication in terms of addition
- Introduction to basic algebraic structures

Should there be additional time in the course, we may pursue other topics by choice of the students. The following topics may be chosen:

- Quadratic reciprocity
- Advanced primality tests and factorization methods
- Sums of squares
- Peano axioms, set theory, and other very low-level foundations of number theory
- *p*-adic numbers (maybe)