Comprehensive Course Syllabus

BC Calculus 1-2

Course Description:

BC Calculus 1-2 covers essentially the same topics as the traditional BC Calculus 1 course and the first half of BC Calculus 2. Homework, assessments, and projects involve working at an abstract, conceptual level. Beyond learning the basic mechanics, students will learn how to think about calculus *mathematically*; that is, students will understand how calculus is formulated as a discipline.

INSTRUCTOR(S):

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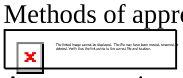
Text(s) / Materials:

Calculus, by Hughes-Hallet et al.

Essential Content:

Understanding Derivatives

- Slope of a tangent line
- Methods of approximation, including notion of step size
- 
- Average vs. instantaneous rate of change
- Geometry of derivatives, including f' and f''
- Graphically constructing derivatives and antiderivatives
- Interpretations in context

Defining the Derivative and theory

- Limits –
  - Intuitive understanding of a limit
  - Computation of basic limits
- Continuity of a function
- Definition of the derivative as a limit

Computations of derivatives

- Rules for differentiating elementary functions
- Product, quotient, and chain rules
- Implicit differentiation

Applications of the derivative

- Position, velocity, acceleration
- Optimization

Differential Equations

- Introduction to modeling quantities that change with time
- General and particular solutions

Applications of the derivative and theory

- Position, velocity, acceleration
- Optimization
- Related Rates
- L'Hôpital's Rule
- Intermediate Value Theorem, Extreme Value Theorem
- Mean Value Theorem

Understanding integrals

- Integral as signed area
- Integral as accumulation of a rate of change
- Basic properties – geometric and algebraic

Defining the Integral and theory

- Approximations of integrals
- Riemann sums
- Definition of an integral
- Fundamental Theorem of Calculus

Computations of integrals

- Antiderivatives of elementary functions
- *u*-substitution

SSLs and Outcomes:

FA: Formally assessed, IA: Informally assessed

IA. Students expected to demonstrate automaticity in skills, concepts, and processes that enable complex thought by

- completing daily homework assignments **FA, IA**
- completing regular assignments **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IB. Students expected to construct questions, forge connections and deepen meaning by

- completing daily homework assignments **FA, IA**
- completing regular assignments **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IC. Students expected to precisely observe phenomena and accurately record findings by

- regularly justifying conclusions and claims in all written work **FA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**
- engaging in daily collaboration to complete or check work **IA**

ID. Students expected to evaluate the soundness and relevance of information and reasoning findings by

- regularly justifying conclusions and claims in all written work **FA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**

- ❖ engaging in daily collaboration to complete or check work **IA**

IIA. Students identify unexamined cultural, historical and personal assumptions and misconceptions that impede and skew inquiry by

- ❖ identifying weaknesses or misconceptions in related prior mathematical concepts **IA**
- ❖ discussing problems from multiple perspectives and opposing views to determine validity to various approaches **IA**
- ❖ engaging in daily collaboration to complete or check work **IA**

IIIA. Students use appropriate technologies as extensions of the mind by

- ❖ exploring mathematical ideas and problem solving using tools such as graphing calculators, Winplot, Mathematica, Excel, etc. **IA**
- ❖ making mathematical conjectures based on graphics and animations **IA**
- ❖ using web-based resources to clarify, verify, or explore ideas **IA**

IIIB. Students recognize, pursue, and explain substantive connections within and among areas of knowledge by

- ❖ applying calculus methods to familiar contexts, such as position, velocity and acceleration, and justifying conclusions **FA**
- ❖ solving problems that require similar means which involve new or less familiar application contexts and justifying conclusions **FA**

IVA. Students construct and support judgments based on evidence through

- ❖ constructing graphs of a function based on the graph of its rate of change, and vice versa, giving full written and/or verbal justification **FA**
- ❖ solving optimization problems, with full justification **FA**
- ❖ exploring and justifying solutions to differential equations **FA**

IVB. Students will be challenged to write and speak with economy, power, and elegance by

- ❖ supporting answers with written justification using precise mathematical notation and language **FA**
- ❖ making sound mathematical verbal arguments using precise language **IA**

VA. Students will identify, understand and accept the rights and responsibilities of belonging to a diverse community by

- ❖ actively participating in class discussions **IA**
- ❖ respecting each others' questions and responses, both in and out of class **IA**
- ❖ collaborating outside of class on Take Home and other assignments without infringing on each others' intellectual capital **IA**

VB. In order for students to make reasoned decisions which reflect ethical standards, and act in accordance with those decisions, students

- ❖ collaborate outside of class on assignments without infringing on each others' intellectual capital **IA**
- ❖ produce their own work on formal assessments **FA**

Instructional Design and Approach:

What makes this course different from the usual BC1-2 sequence? Briefly, the intent is to train students to think mathematically. There is a strong emphasis, especially as students move into the BC2-3 course, on rigorous proof in assignments. More difficult topics – such as hyperbolic trigonometric functions – are routinely addressed. Questions, even if tangential to the current topic, are discussed in class. Mathematicians do wonder what happens next, and this sense of wonder is indulged.

Students learn that there are usually no solutions to exams or homework assignments (other than the problems in the book). They keep working, investigating; when mathematicians explore new problems, they are *new*, without answers conveniently located at the end of the book. Thus students get a real sense that they get out of the course what they invest, as is certainly true when delving into mathematical ideas. Learning becomes more student-driven and less teacher-driven.

Assignments:

Homework: There will be regular, if not daily, homework assignments. Do them. Students falling behind in their homework will stay after school to complete it.

In-class assessments: There will be exams every other Friday. Each exam will be in two parts. The first will consist of an assessment of basic skills; it is expected that most students will earn an A on this half of the exam. The second part will consist of more open-ended questions. Some will be rather straightforward, while others will be difficult. It is not expected that you will have time to work on every problem on this part of the exam. For very difficult problems, it is not necessarily important that you get them completely correct. Certain problems may be included to see how you approach a particular problem -- what tools you bring to bear on a particular problem.

Writing assignments/video projects/presentations: You may be expected to be able to explain ideas in writing, through video, or more elaborate presentations. These will be assigned periodically.

At any time during the semester, you may propose your own writing assignment or project. You must write up a brief proposal, and it must be approved by the instructor. As a result, not all students will necessarily be doing the same assignments.

Student Expectations:

It is important to realize that what is known as the ``BC1-2 Fast Track" is not simply a matter of covering the usual BC1-3 curriculum but moving at 3/2 the pace. What distinguishes BC1-2 students is

1) A willingness and ability to practice routine problems outside of class. Concepts will be covered during class, while more focused practice will be done as homework;

2) An enthusiasm for open-ended questions and intellectual curiosity. In-class inquiries focus on *discovery* of important ideas, rather than just reading them in the textbook;

3) The daring to explore ideas with technology – *Mathematica* and Excel especially – realizing that most applied problems involve data sets which are not easily modeled by a single, simple function;

4) The discipline to practice and learn independently; students must consistently perform at a B level; otherwise, they will be re-placed in BC1.

Assessment Practices, Procedures, and Processes:

Expectations for:

A: Very few problems with mechanics, along with a demonstrated ability to solve conceptually challenging problems. In order to earn an A, you must hand in additional work which extends or generalizes work done in class. For example, consider the trigonometry problems on the Warmup handout. On your own initiative, you replace all circular trigonometric functions with hyperbolic trigonometric functions, and see if the problems can be solved. In addition, you note what differences arose from this extension, and any problems encountered in solving the altered problems, writing your observations in paragraph form. You may hand in such extensions at any time over the course of the semester.

A-: Very few problems with mechanics, along with a demonstrated ability to solve conceptually challenging problems.

B/B+: Occasional or regular problems with mechanics, with some issues regarding solving conceptually challenging problems.

B-: Indicates that the student is not meeting course expectations.

These expectations are purposely somewhat vague – it is expected that students are more concerned with learning calculus than with your grade in the course. No student enrolled in the course for the entire semester will receive a grade lower than a B.

Your grade for each quarter will be roughly a B if fewer than one-fourth of your assessments are A's, a B+ if up to one-half of your assessments are A's, an A- if up to three-fourths of your assessments are A's, and an A if more than three-fourths of your assessments are A's and your extensions are sufficiently well done. There is some latitude in assigning grades; especially inspiring participation in class, for example, could influence your grade.

Students not performing at a B level will be moved to the traditional BC 1 class. After the first assessment which is below a B level, you will meet with the instructor. After the second assessment, you will move to the traditional BC 1 class.

The two components to your grade will be the individual in-class assessments (75—85%) and projects (15—25%). For semester grades, each quarter is weighted 40%, while the Final Assessment is weighted 20%. The number in PowerSchool is not absolute; students may keep a portfolio of their work which will be taken into account at the end of the semester.

Sequence of Topics and Activities

Week 1

Sections 1.1, 1.2, 1.3, Appendix A (calculator stuff)
Inequalities and distance/ Graph vocabulary

Sketching common functions

Week 2
Rate of Change 4
Euler 1-3 (by hand and program)
Start Section 1.4 – derivative as instantaneous rate of change and as slope.
Position, velocity, acceleration relationship

Week 3
Section 1.5 - Zooming to approx. tangent (local linearity); symmetric difference - Nderiv on calcs
Sections 1.6 (Geometry of Derivatives)
1st Derivative Test (sign chart)

Week 4
Slopes 2, Section 1.7
Second Derivative Test
Section 2.1 – formal definition of derivative at a point

Week 5
Section 2.2 – formal definition of derivative function
Limits 1-4
Sections 2.3 (start)

Week 6
Limits 5-7
Sections 2.3 - 2.5 (start)

Week 7
Section 2.5 (finish), Section 2.6
Derivatives of Exponential Functions handout
$\ln(x)$ & $\log_b(x)$ derivatives
Section 2.7 Section 3.1 (start)

Week 8
Section 3.1 (finish), Section 3.2
Section 3.3

Week 9
Section 3.4 and Section 4.3 (Optimization)

Week 10
Sections 4.3 & 4.6

Week 11
Section 4.1: Diff Eqs and Slopefields
Slopefield worksheets; Newton's Law of Cooling
Start Section 4.2: Limits involving infinity &
Composites; L'Hôpital's Rule (straightforward)

Week 12
Finish Section 4.2: L'Hôpital's Rule (harder)
Section 4.4 (Parametrics); Parametrics 1 & Parametrics 2 worksheets

Week 13
Section 4.8 (IVT&EVT);
Section 4.9 (MVT)

Week 14
Section 5.1 (start of work on integrals)
Euler Encore 1, 2 & 3 worksheets
Sections 5.2 & 5.3
Integrals, antiderivatives, FTC

Week 15
Finish Section 5.4.
Section 5.5: Reading integral tables, completing the square, reduction formulas

Week 16
Approximations 1-4; Sections 5.6 & 5.7
Definition of definite integral via limits
Riemann Sums, LH, MP, RH, Trap. Approx.
Integrals and Approximation Practice 1