Comprehensive Course Syllabus

Mathematical Investigations II

Course Description:

Mathematical modeling is the process of creating a mathematical representation of some phenomenon in order to gain a better understanding of that phenomenon. This course is designed to show the interconnectedness between the study of math and the application of math to various fields. This course will encompass many academic disciplines including life sciences, engineering, physics, economics, and finance. This course will be an investigation of meaningful and realistic problems and how a mathematical approach overlays each of these areas. Most of the mathematics currently seen and studied was developed several hundred years ago, and we will be seeing how math is being used and developed more recently.

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INSTRUCTOR(S):

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Text(s) / Materials:
Texts available on Springerlink via IMSA IRC for free download of chapters.
Other materials will be made available as needed.
Students will be expected to type all papers in LaTex and submitted in pdf.
Students are required to have a graphing calculator. TI-89 Titanium is recommended.
Students will need their laptops daily for online research and presentation work.

Essential Content:

- Connections - This course will connect Mathematics and Science. Applications to biology, engineering, and physics will be included.
- Real World Connections - Real data will be collected and statistically analyzed. Students will also have the opportunity to collect data as well.
- Conceptual understanding - Students will develop conceptual understanding of real world phenomena.
- Programming Skills - Students will learn programming skills that apply to real world scenarios.
- Open Problems - Students will learn that not every problem has an easy and elegant solution. Students will think that all mathematics and science has been discovered. In this course, the student will discover that each field has a list of open problems and conjectures that need to be studied from multiple perspectives.
- Multiple Perspectives - Students will encounter problems from different points of view. They will be analyzing data, interpreting results, looking at verbal descriptions that they must turn into a problem to be solved, and they will be using a variety of points of understanding. For instance, they will formulate a problem, use statistics on a set of data, numerical solve problems without analytic solutions, and produce plots of data and systems of nonlinear equations.

IMSA Standards addressed:

I.B IMSA students construct questions which further understanding, forge connections, and deepen meaning.

Students recognize intellectual discomfort when presented with anomalies, as well as new and problematic situations. They search for coherence and consistency of ideas as they try to integrate diverse elements, relations and values within and between disciplines, and within themselves.

Students form powerful and significant questions which extend their capacity to order and clarify experiences and make them intelligible. Through these questions, students reveal knowledge and insights; these questions may also lead to new insights.

I.C IMSA students precisely observe phenomena and accurately record findings.

Students, as observers, perceive, study, or examine anything that can be described or appraised through the senses, either directly or indirectly. In doing so, students ascertain the extent, quantity, dimensions, or capacity of this object, fact, circumstance, or experience. Students use a method or procedure as an established guide for action and agreement that is appropriate to the task and to the context. This context may be mathematics, the sciences, the humanities, the arts, or residential life.

From this observation, students create a record that serves to preserve the description or appraisal as evidence or as an account. This record could take the form of a sketch book, laboratory notebook, journal, videotape, audio recording, etc. It also conforms to established standards and may lead to common understandings between and among students and other individuals.

I.D IMSA students evaluate the soundness and relevance of information and reasoning.

Students examine and appraise data, facts, and processes, as well as draw inferences or conclusions from these sources. Examination entails ascribing value in terms of self-defined and externally defined standards of usefulness or importance. In order to achieve soundness, one must consider validity, reliability, justness and believability. Relevance involves consideration of pertinence, applicability, and suitability. The conditions under which this evaluation of soundness and relevance occur include: investigation (structured and methodical); exploration (focused, open-ended, and observation dependent); and interpretation (analytical and inferential).

II.B IMSA Students find and analyze ambiguities inherent within any set of textual, social, physical, or theoretical circumstances.

A dynamic tension or an organic form is the essence of all thought. As suggested in SSL-III.B, all facts, observations, abstractions, and sets of data do not exist alone or in a vacuum. Rather they can only be understood and have meaning when interpreted within the complex web of political, social, economic, and historical relationships from which they have evolved.

Thus, ambiguity, uncertainty, or vagueness are inherent in every set of textual, social, physical, or theoretical circumstances that we encounter in our attempt to construct meaning. The failure to recognize, analyze, and understand these ambiguities results in an inability to construct meaning. We are then left with the sole option of believing whatever we are told rather than the more empowering circumstance of constructing valid meaning for ourselves through the recognition and analysis of omnipresent ambiguities.

III.A IMSA students use appropriate technologies as extensions of the mind.

Students use technologies to survey, explore, model, manipulate, focus, and communicate. In doing so, they recognize the scope, limitations, and appropriateness of these technologies. These include, but are not limited, to technologies available today. Currently, technologies range from print-based to computer-based technologies; from word processing to modeling software; from graphing calculators to super computers; from still-frame photography to DVD and multi-media; from microscopes to mass spectrometers; from telephones to the World-Wide Web.

Future leaders must be able to navigate an ever-expanding sea of investigative possibilities. Whether searching for existing information or constructing new knowledge, technological confidence and competence are essential. This includes a mastery of various types and models of equipment and software. These skills serve as a foundation that facilitates positive transfer to new and emerging technologies that take students beyond classroom walls.

In addition, students learn essential information-management strategies in order to effectively search for relevant, valid, and reliable information. Students demonstrate appropriate critical thinking and problem-solving skills as they access, evaluate, compose, store, and share information. Their products demonstrate an integration of information sources and ways of knowing, as well as innovative presentation possibilities

III.B IMSA students recognize, pursue, and explain substantive connections within and among areas of knowledge.

Students seek out the interrelationships of ideas from diverse contexts. Interrelationships construct a web of connection within and among disciplines and life experiences. Connections serve as powerful and flexible means to solve problems and to acquire a deeper appreciation for the consistency and beauty of areas of knowledge. These connections can be expressed by means of structures such as concepts, themes, or significant issues or problems which enhance understanding.

Connections are substantive when they disclose fundamental relationships by revealing similarities, contrasts, logical, or causal relationships; apply broadly and pervasively in multiple contexts; and provide an element of fascination to provoke and sustain intellectual interest. Substantive connections enable the learner to construct integrated wholes and to construct meaning in multiple contexts.

Each of the Standards of Significant Learning provides a window for capturing evidence of understanding connections of important ideas within and among academic disciplines and other life situations. Students demonstrate their understanding of interrelationships in multiple ways.

III.C IMSA students recreate the beautiful conceptions that give coherence to structures of thought.

Sense can be made of a work only if its parts are connected in a meaningful way. The extant connections are in themselves so startling, mysterious, and evocative that they produce an aesthetic response in those with deep understandings. Such conceptions are beautiful to the extent that their elegance, simplicity, or power produce an aesthetic response in the learner. A beautiful conception provides the knower with a feeling of seeing past the surface structure to the wellsprings of a phenomena.

IV.A IMSA students construct and support judgments based upon evidence.

After precisely observing phenomena and accurately recording findings, students piece together critical and necessary documentation to form judgments. Students evaluate the completeness, accuracy, and quality of the data; and from this data, they create and articulate reasoned decisions.

Students begin to identify norms, values, and criteria inherent within the judgment being made. They cite evidence to support or refute various possibilities and consequences of the impending judgment. Also, they distinguish between preferred and less preferred procedures. Their reasoned decisions and connected support structures constitute the judgment. It should be noted that sometimes learners arrive at a conclusion which is misleading, incomplete, or inadequate. At this point they need to revisit their earlier thinking.

IV.B IMSA students write and speak with power, economy, and elegance.

Power: To write and speak with power is demonstrated by an ability to influence others by constructing and delivering compelling, accurate, and forceful oral and written communication grounded in truth and appropriate for the audience.

Economy: To write and speak with economy is demonstrated by focused, organized, and parsimonious oral and written communication. Statements are constructed to avoid excess. This implies careful planning, appropriate vocabulary, efficient use of time and space, and an appreciation of the purpose of the communication.

Elegance: To write and speak with elegance is demonstrated through oral and written communication that exhibits clarity, richness, and a graceful style. The communication is direct, free of overstatement and embellishment. This allows for both the author and the audience to be true to their voice.

In general students' communications:

Get to the point.

Stimulate insight.

Make an impact.

Do this with distinction

IV.C IMSA students identify and characterize the composing elements of dynamic and organic wholes, structures, and systems.

This standard deepens the experience of ideas, works of art, theoretical models, and organisms, to move students beyond "rigid application of algorithm" into an experience of internalized performance. For example, in order to achieve deeper meaning or knowledge and affect in music, students need to be able to identify and characterize melody, harmony, rhythm, tone color, texture, and form. The music critic Carl Meyer asserts that Embodied musical meaning is... a product of expectation. A listener who has considered the work in the way in which a performer would, anticipates the listening experience. This listener becomes one with the music in the listening. To listen without this prior experience is a more superficial endeavor.

Similarly, the particular, temporal elements of a specific mathematical problem need to lead students to an experience of the generalization, formula, or theory it represents.

IV.D IMSA students develop an aesthetic awareness and capacity.

Aesthetic awareness is the capability of distinguishing the sensuous, emotional, or intellectual from the aesthetic: Students recognize that experiences are never wholly intellectual or emotional, but rather a mixture or a marriage of the two. It is this combination that brings to bear the tools necessary for aesthetic literacy. As in critical thinking, meaning is deepened via analysis, synthesis, and evaluation. Further analysis and re-evaluation leads students to valuing and intellectual/emotional ownership.
In recognizing this we develop aesthetic capability, and are therefore able to more readily find, and create things of beauty.

Common Core Standards-
N-Q: Reason quantitatively and use units to solve problems.

2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement
when reporting quantities.

A-SSE: Seeing Structure in Expressions
Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED: Create equations that describe numbers or relationships.
F-IF: Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.

F-IF: Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Instructional Design and Approach:
Students are expected to develop skills that allow them to solve problems from the real world in various fields of application such as economics, biology, and physics.
They will learn how to formulate a problem, define the variables and parameters, analyze data, and produce graphical analysis of such topics.
They will learn how to analyze data from statistical perspectives in R.
Students will learn to model systems using linear programming techniques, differential equations (from a numerical perspective), and Markov Chains.
They will present a verbal description of the results from their systems of equations.
Further they will make connections to what is known and what is not known. They will develop the state of the art.
Lastly, they will seek to formulate conjectures and expand the ideas to further the field.

ASSESSMENT PRACTICES, PROCEDURES, and PROCESSES:
Grades will be apportioned as follows:

- Weekly notebook containing worksheets, summaries of journal articles, and annotated bibliographies 30%
- 1st Quarter Project 20% Student will select a topic regarding modelling from a biological perspective to present a 25 minute presentation to the class along with a research paper. This will be done in teams or partners.
- Final Project 20% Student will select a topic regarding stochastic processes or graph theory
- Quizzes 20% Students will have quizzes regarding mathematical models from a numerical and conceptual viewpoint. Students will also have short quizzes over the presentations presented during class.
- Classroom Participation 10% Students will lead in class discussions, complete activities, and post in the online discussion forum.

Grading scale:

- A - 90% or above
- B - 80% or above but less than 90%
- C - 70% or above but less than 80%
- D - less than 70%

Sequence of Topics and Activities

- First 3 weeks- intro to/ linear- graph theory (2 days) transition between intro and life sciences (adjacency matrices)
  - Life Science Applications- 4-6wks
    - Epidemiology- SI model/SIR model- they come up with an application
    - Species Competition
    - Population Growth (careful with BC topic coverage)
    - Enzymatic- Michaelis-Menten kinetics
  - Game Theory/ Evolutionary Game Theory- 4wks
    - Prisoner's Dilemma
    - Tit for tat, tit for two tats- always cooperate always defect
    - Hawk Dove
    - Economic Applications
  - Physics/Mechanics applications- students will be gathering their own data having conducted experiments- 4 wks
  - Stochastics-individual projects ending in presentations for the final- Markov chains- monopoly problem (Mathematica), casino problems, random walks, weather, or topic of self-selection, Gillespie algorithm, economic applications