# Comprehensive Course Syllabus
**Modern Geometries**
**Spring 2018**

**Course Description:**

Modern Geometries aims to expose students to rich geometric ideas that go beyond the Euclidean plane. Topics will include axiom systems, metric spaces, curvature, projections, inversions, duality, constructions, symmetry, tilings, and solid geometry.

INSTRUCTOR(S):

Dr. Micah Fogel
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Office Hrs: 11:00am-1:15pm in A157

**Meeting Days, Time and Room(s)**
A155, Mod 7, A–D days

**Text(s) / Materials:**

Students will simply need a notebook to house and organize materials. Projects will require materials such as posterboard, markers, scissors, glue, and the like.

This course does not employ a textbook but from time to time textual material may be offered to support the students' investigations.

The following books are recommended for supplemental support.

*Famous Problems of Geometry and How to Solve Them*, Benjamin Bold
*Introduction to Non-Euclidean Geometry*, Harold E Wolfe
*Mathematical Cranks*, Underwood Dudley
*Geometry & Symmetry*, L C Kinsey, T E Moore, & E Prassidis

In addition to books, students are expected to have a laptop computer to run applications and a straightedge and compass. A protractor may also be helpful.

**Essential Content:**

The theme of Modern Geometry will be to explore new geometric models through a Euclidean lens: by more carefully examining our well-known axioms, we find other equally valid geometric systems that are important in their own right. The essential content will be to challenge classical Euclidean interpretations with new models:

(1) Use metric spaces to challenge Euclidean distance, as in taxicab geometry.

(2) Use curvature to challenge the parallel postulate, as in spherical and hyperbolic geometry.
(3) Use inversions and duality to challenge definitions, like point and line.
(4) Use constructability to challenge continuity, as in the case of non-constructible numbers.
(5) Use solid geometry to explore more recent ideas of compactness and dimensionality.

Occasional in-depth explorations of a topic may also occur throughout the course, such as distinguishing knots through relatively simple (to calculate) invariants.

The content of the course will primarily meet departmental standards I and D (geometry and intra-disciplinarity), the process meets departmental standard B (deductive structures), and the assessments meet departmental standard C (justification and writing). As a byproduct of our choice of topics, this course will help to reinforce ideas from precalculus as they are applied in novel settings. For example, roots of polynomials are vital to constructability, vectors are needed to compute angles in spherical geometry, and trigonometry is essential for solid geometry and spherical triangles.

**Topics Covered:**

This course will proceed through multiple units.

- Spherical geometry and trigonometry
- Taxicab (and related) geometry
- Neutral Geometry
    - Euclid's definitions and axioms
    - Propositions 1 through 28 of *Book I of The Elements*
- Nonneutral Geometry
    - Propositions 29 through 32 of *Book I of The Elements*
    - The Parallel Postulate and its consequences
    - The Pythagorean Theorem
    - Similarity
    - Structure of circles and polygons
- Hyperbolic and Elliptic Geometry
    - Hyperbolic Postulate on parallelism
    - Hyperbolic distance and models of the hyperbolic plane
    - Identification of antipodal points on a sphere and elliptic geometry
- Other Geometries compatible with neutral geometry
    - Perspective drawing and projective geometry
    - Finite geometries
- Constructions
    - Compass and straightedge constructions
    - Constructible numbers and polygons
    - Impossible constructions
    - Changing the rules (collapsing compass, Mascheroni, Steiner-Poncelet, neusis, and solid constructions)

        - Axioms of origami
        - Foldable polygons
        - Folding impossible constructions
    - Topology
        - Euler's formula
        - Map coloring
        - Knots

**SSLs and Outcomes**
The main SSLs to be addresses in this course are:

I.A Develop automaticity in skills, concepts, and processes that support and enable complex thought.
*Students will learn and apply formulas and theorems to facilitate explorations into more advanced topics.*

I.B Construct questions which further understanding, forge connections, and deepen meaning.
ID. Evaluate the soundness and relevance of information and reasoning.
II.A Identify unexamined cultural, historical, and personal assumptions and misconceptions that impede and skew inquiry.
*Students will reconsider assumptions implicit in geometric conceptions in order to better understand cause and effect.*

III.A Use appropriate technologies as extensions of the mind.
*Students will use models, digital and otherwise, to better visualize alternative geometries.*

IV.A Construct and support judgments based on evidence.
*Students will write rigorous proofs using the axioms in their particular geometry.*

**Instructional Design and Approach:**

This course will be centered on exploration of non-Euclidean geometries. The introduction to the course will use lecture to reiterate familiar facts of Euclidean geometry, but with a new emphasis on the axioms they require. The course will then transition into the non-Euclidean realm. Students will learn through inquiry-based group activities in class, and important ideas will be developed by students during in-class work and nightly homework assignments. Throughout the semester, students will give oral presentations, write reports, and build models with some personal choice permitted.

**Student Expectations:**

Students will be expected to be present and attentive in class, diligent and timely in completing homework, and inquisitive and industrious during group activities. The student must make arrangements for absences or missed work ahead of time. As always, collaborative work is encouraged, but copying and other forms of academic dishonesty are strictly prohibited.

**Assessment Practices, Procedures, and Processes:**

Like the course itself, the assessments emphasize exploration into alternative geometries. A high percentage of the course grade is based on homework, projects, and presentations. There will still be a quiz at the end of each unit, although there is no final exam.

Homework will be given on a nightly basis, although will only be collected and graded roughly once per week.

The grading scale for the course is:

- 33% Homework
- 33% Participation
- 33% Projects

**Suggested Sequence and Timeline of Topics and Activities:**

| ACTIVITY | DURATION |
|---|---|
| **Spherical geometry** | 1 week |
| **Taxicab geometry** | 1 week |
| **Neutral geometry** | 1 week |
| **Nonneutral Geometry** | 1 week |
| **Hyperbolic and Elliptic geometry** | 1 week |
| **Projective and finite geometries** | 2 weeks |
| **Constructions** | 2 weeks |
| **Topology** | 2 weeks |

Times have been underestimated, so the above material will take more than the allotted eleven weeks. Student may also find interesting avenues to explore to which we will devote any needed time. Thus, the material will easily expand to fill the entire course. If there is extra time toward the end of the semester, students will be given class time to work on their final projects.