Comprehensive Course Syllabus

Theory of Analysis

Course Description:

Theory of Analysis surveys the topics in a college "Introduction to Real Analysis" or "Advanced Calculus" class. The main focus of this class is on the nature of analytic proof. Analytic proof will be developed as the class covers the standard material from a one semester college course in analysis. Topics include a rigorous treatment of limits, sequences, continuity, and differentiation.

Text(s) / Materials:

This course has no textbook. Extensive course notes are posted on the class web page.

INSTRUCTOR(S):

- Name(s): Micah Fogel
- Office Number(s) (When and where you are available for help.): A157

  Office Hours:

  Mod 2 every day; mod 8 A/C days; mods 4 and 5 twice each week (days vary)

- Telephone number(s): (630) 907-5086
- Email address(es): fogel@imsa.edu

Meeting Days, Time and Room(s)

Mod 3, A155

Essential Content:
Content is that which is typically covered in a one semester, junior/senior level college Real Analysis course. See last page for approximate time spent on each topic.

SSLs and Outcomes:
FA = Formally assessed, **IA** = Informally assessed

IA. Students expected to demonstrate automaticity in skills, concepts, and processes that enable complex thought by

- completing daily homework assignments **FA, IA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**
- presenting solutions to problems in class **FA**
- presenting theory to class in group presentations **FA**

IB. Students expected to construct questions, forge connections and deepen meaning by

- completing daily homework assignments **FA, IA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**
- preparing for group presentations **IA**

IC. Students expected to precisely observe phenomena and accurately record findings by

- regularly justifying conclusions and claims in all written and oral work **FA,IA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions and presentations **FA, IA**
- engaging in daily collaboration to complete or check work **IA**

ID. Students expected to evaluate the soundness and relevance of information and reasoning findings by

- regularly justifying conclusions and claims in all written work **FA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA, FA**
- engaging in daily collaboration to complete or check work **IA**
- preparing for group presentations **IA**

IIA. Students identify unexamined cultural, historical and personal assumptions and misconceptions that impede and skew inquiry by

- discussing problems from multiple perspectives and opposing views to determine validity to various approaches **IA, FA**
- engaging in daily collaboration to complete or check work **IA**
- preparing for group presentations **IA**

IIIA. Students use appropriate technologies as extensions of the mind by

- exploring mathematical ideas and problem solving using tools such as graphing calculators, Winplot, Mathematica, Excel, etc. **IA**
- making mathematical conjectures based on reasoned exploration **IA, FA**

IIIB. Students recognize, pursue, and explain substantive connections within and among areas of knowledge by

- applying analytical methods to familiar contexts, e.g. proving well known theorems from BC Calculus **FA**

- ❖ solving problems that require similar means which involve new or less familiar application contexts and proving conclusions **FA**

IVA. Students construct and support judgments based on evidence through

- ❖ experimenting with 3d graphs then generalizing structure **FA,IA**
- ❖ hypothesizing and proving properties of real-valued functions **FA**
- ❖ exploring, justifying, and presenting solutions to problems in class on a daily basis **FA**

IVB. Students will be challenged to write and speak with economy, power, and elegance by

- ❖ supporting answers with written justification using precise mathematical notation and language **FA,IA**
- ❖ making sound mathematical verbal arguments using precise language **FA,IA**
- ❖ presenting solutions to problems to the class **FA**
- ❖ making group presentations on the theory of analysis to the class **FA**

IVC. Students will identify and characterize the composing elements of dynamic and organic wholes, structures and systems.

- ❖ actively developing the theory of analysis **FA,IA**
- ❖ preparing for group presentations **IA**

IVD. Students will be challenged to develop an aesthetic awareness and capability.

- ❖ looking at the historical development of analysis **IA**
- ❖ Comparing student solutions and discussing relative merits, including elegance **FA,IA**
- ❖ completing daily homework assignments **FA, IA**
- ❖ engaging in daily collaboration to complete or check work **IA**
- ❖ completing quizzes and tests **FA**

VA. Students will identify, understand and accept the rights and responsibilities of belonging to a diverse community by

- ❖ actively participating in class discussions **IA**
- ❖ respecting each others' questions and responses, both in and out of class **IA**
- ❖ collaborating outside of class on group presentations and other assignments without infringing on each others' intellectual capital **IA**

VB. In order for students to make reasoned decisions which reflect ethical standards, and act in accordance with those decisions, students

- ❖ collaborate outside of class on assignments without infringing on each others' intellectual capital **IA**
- ❖ produce their own work on formal assessments **FA**

Instructional Design and Approach:

Students should be involved in exploration of the concepts and topics through reading of the text and outside material, giving presentations to classmates, through preparing for group presentation and then presenting new topics in the theory to their classmates, and solving problems in individual and group settings. Students will be asked to engage in the dialogue of problem solving, and to help their classmates understand the content of the course. Learning to write and speak in clear and precise mathematical language is a major goal throughout this course.

Student Expectations:
Students are expected to come to each class prepared to contribute to the classroom learning experience. This involves reading all assigned text and attempting all assigned problems before coming to class. Students are expected to work together and independently in deepening their understanding of course concepts. Students will have to take careful and complete notes in class, as the text does not cover all required material. Students will be expected to work in groups to make classroom presentations on selected topics hroughout the semester.

Assessment Practices, Procedures, and Processes:
Students are assessed through a variety of means including, but not limited to: written in-class and out-of-class exams, quizzes, individual classroom presentations, group classroom presentations, and homework.

Quarterly Grades:	Homework	- 30%
	Problem Presentation	- 30%
	Exams & Quizzes	- 40%
Semester Grades:	Semester Exam	- 20%
	Work during the semester	- 80%

Sequence of Topics and Activities

Week 1:
Preliminaries
Intro to LaTeX, intro to proofs and writing. Notation we need to know.

Week 2:
Meet the real numbers
Axioms for the real numbers. Properties of the real numbers. Basic theorems and relation-ships.

Week 3:
Introduction to sets
Notation and operations. Functions and relations. Finite and infinite; countable and un-countable sets.

Weeks 4–5:
Basics of point-set topology
Open and closed sets. Accumulation and adherent points. Metric spaces. Compactness.

Weeks 6–7 :
Limits and Continuous Functions
Convergent sequences. Cauchy sequences. Limits of functions. Continuous functions. EVT. Connectedness and IVT. Uniform continuity.

Weeks 8–9:
Differentiation
Definition and properties. Algebra and the chain rule. MVT.

Week 10:
Taylor series
Taylor's formula with remainder.

Week 11-13:
Integration
Riemann and Riemann-Stieltjes integrals. Basic properties. Integration by parts. Change-of-variables. Fundamental Theorem.

Week 14-15:
Sequences of functions
Pointwise and uniform convergence. Differentiation and integration.
Week 16:

Back to the real numbers
Constructing the real numbers from simpler objects.
Week 17:

More advanced topics
Henstock/Kurzweil/Denjoy integral, consideration in multiple variables, complex numbers and analysis, Lebesgue integral, implicit function theorem, . . .