Comprehensive Course Syllabus

Algebraic Structures I and II

Course Description:
The topic of this course is either Linear Algebra or Abstract Algebra. The majority of the time the topic will be linear algebra, the abstract option occurs in rare years when a large number of students took the linear option as juniors the year before and need another advanced course.

OPTION 2 (Abstract Algebra)
The content of this course is flexible, but is generally an introduction to abstract algebra. Students learn about groups, subgroups, homomorphisms, and the structure of various groups (such as the structure theorem for finitely generated Abelian groups, the Sylow theorems, etc.) Students also investigate the basics of rings. Ring topics include ideals and homomorphisms; PIDs, UFDs, and Euclidean domains; fields and (time permitting) field extensions including applications such as constructibility. All aspects of the course are presented with full mathematical rigor, and students are expected to produce proofs of equivalent quality to mathematics majors at a university.

INSTRUCTOR(S):

- Name(s): **Micah Fogel**

- Office Number(s) (When and where you are available for help.): **A157**
Daily 9AM – 11AM, 1:15PM – 2:15PM

- Telephone number(s): **x-5086**

- Email address(es): **fogel@imsa.edu**

Meeting Days, Time and Room(s)
Mods 5 Daily

Text(s) / Materials:
(Abstract option) Judson, Thomas W.. *Abstract Algebra: Theory and Applications (Annual Edition 2018).* GNU Free Documentation license downloadable at abstract.pugetsound.edu. Copyright 1997 – 2018.

Essential Content:

Groups

- Definition
- Cyclic groups
- Permutation groups
- Subgroups and cosets
- Isomorphism
- Homomorphism and factor groups

Rings

- Definitions
- Ideals and homomorphisms
- Factorization theory

Fields

- Polynomial rings over fields
- Extension fields

SSLs and Outcomes:

FA = Formally assessed, IA = Informally assessed

IA. Students expected to demonstrate automaticity in skills, concepts, and processes that enable complex thought by

- completing daily homework assignments **FA, IA**
- completing regular problem sets **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IB. Students expected to construct questions, forge connections and deepen meaning by

- completing daily homework assignments **FA, IA**
- completing regular problem sets **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IC. Students expected to precisely observe phenomena and accurately record findings by

- regularly justifying conclusions and claims in all written work **FA,IA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**
- engaging in daily collaboration to complete or check work **IA**

ID. Students expected to evaluate the soundness and relevance of information and reasoning findings by

- regularly justifying conclusions and claims in all written work **FA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**
- engaging in daily collaboration to complete or check work **IA**

IIA. Students identify unexamined cultural, historical and personal assumptions and misconceptions that impede and skew inquiry by

- identifying weaknesses or misconceptions in related prior mathematical concepts **IA**
- discussing problems from multiple perspectives and opposing views to determine validity to various approaches **IA**
- engaging in daily collaboration to complete or check work **IA**

IIIA. Students use appropriate technologies as extensions of the mind by

- exploring mathematical ideas and problem solving using tools such as graphing calculators, Winplot, Mathematica, Excel, etc. **IA**
- making mathematical conjectures based on reasoned exploration **IA**

IIIB. Students recognize, pursue, and explain substantive connections within and among areas of knowledge by

- applying calculus methods to familiar contexts, e.g., applying green's Theorem in physics **FA**
- solving problems that require similar means which involve new or less familiar application contexts and justifying conclusions **FA**

IVA. Students construct and support judgments based on evidence through

- experimenting with 3d graphs then generalizing structure **FA,IA**
- Hypothesizing and proving vector properties **FA**
- exploring and justifying solutions to problems in class on a daily basis **FA**

IVB. Students will be challenged to write and speak with economy, power, and elegance by

- supporting answers with written justification using precise mathematical notation and language **FA,IA**
- making sound mathematical verbal arguments using precise language **FA,IA**

IVC. Students will identify and characterize the composing elements of dynamic and organic wholes, structures and systems.

- Actively developing the theory of vector calculus **FA,IA**

IVD. Students will be challenged to develop an aesthetic awareness and capability.

- Looking at the historical development of vector calculus **IA**
- Comparing student solutions and discussing relative merits, including elegance **IA**

VA. Students will identify, understand and accept the rights and responsibilities of belonging to a diverse community by

- actively participating in class discussions **IA**
- respecting each others' questions and responses, both in and out of class **IA**
- collaborating outside of class on Take Home and other assignments without infringing on each others' intellectual capital **IA**

VB. In order for students to make reasoned decisions which reflect ethical standards, and act in accordance with those decisions, students

- collaborate outside of class on assignments without infringing on each others' intellectual capital **IA**
- produce their own work on formal assessments **FA**

Instructional Design and Approach:

This course is taught in the nature of a university-level mathematics course. Thus the bulk of class time is lecture. Frequent homework familiarizes students with the key ideas, but students are expected to do problems well in excess of those assigned if they expect to become thoroughly knowledgeable about the material. Such extra problems are suggested but not collected as homework. The idea is to get students used to the pattern of math instruction they will see as a math major at university: lecture, note-taking, reading ahead in the text, selecting one's own practice problems to become conversant with material.

Student Expectations:

This course is taught in the same manner as an upper division course at a university would be taught. Expectation of students will be the same as for university students: compelte solutions to problems, accurate use of terminology and notation, on-time submission of assignments, students take responsibility for learning. This is not a course for inexperienced students who need their hands held! Workload is high and expectation for mathematical advancement is high.

Assessment Practices, Procedures, and Processes:

There are approximately one-to-two homework assignments per week, consisting mostly of problem from the text which are to be completed with high expectations for mathematical rigor. There will be quizzes every few weeks. Students will never be asked to take a quiz on material for which they have not yet had homework returned and for which they had opportunity to ask questions. Every assignment lists required problems and recommended problems; students should complete many of the recommended problems if they expect to get the most out of the class.

Sequence of Topics and Activities

Abstract Option
Weeks 1 – 2: Preliminaries, basic vocabulary and notation, motivating examples
Weeks 2 – 3: Groups: basic definitions, examples, terminology
Weeks 4 – 5: Subgroups, cosets, counting
Weeks 5 – 6: Isomorphisms and homomorphisms
Weeks 7 – 8: Structure theory
Weeks 9 – 10: Rings: basic definitions, examples, terminology
Weeks 10 – 11: Homomorphisms and ideals
Weeks 11 – 13: Special types of rings—UFD, PID, Integral domain, field, skew-field. Polynomial rings and factorization.
Weeks 14 – 15: Fields and extensions
Week 16: Applications and tying up loose ends.