Comprehensive Course Syllabus

Multivariable Calculus

Course Description:

Students will apply and extend their knowledge of calculus to problems involving several variables. They will examine the similarities and differences between one- and several-variable situations from computational, geometrical, and theoretical points of view. The course covers the material from a traditional semester-long university course: geometry and algebra of vectors, coordinate systems, functions of several variables and their graphs and behaviors, differentiation (partial derivatives, gradients, divergence, curl, etc.), and integration (multiple integrals, path, line, and surface integrals). Other topics may include applications (min/max problems, work, flux, etc.), Change-of-Variables Theorem, Green's Theorem, Gauss' Theorem, and Stokes' Theorem.

INSTRUCTOR(S):

- Name(s): **Micah Fogel**

- Office Number(s) (When and where you are available for help.): **A157,**

Hours: daily 9 a.m. – 11 a.m., 1:15 – 2:15 p.m.

- Telephone number(s): **x-5086**

- Email address(es): **fogel@imsa.edu**

Meeting Days, Time and Room(s)

Section 1: A155, ABCD days, Mod 7
Section 2: A155, ABCD days, Mod 8

Text(s) / Materials:

Colley, Susan J. (2003). *Vector Calculus*, 4th ed. Pearson.

Essential Content:

Content is that which is typically covered in a one semester college multivariable calculus course. (Not including linear algebra or differential equations material.)

SSLs and Outcomes:

FA = Formally assessed, IA = Informally assessed

IA. Students expected to demonstrate automaticity in skills, concepts, and processes that enable complex thought by

- completing daily homework assignments **FA, IA**
- completing regular problem sets **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IB. Students expected to construct questions, forge connections and deepen meaning by

- completing daily homework assignments **FA, IA**

- completing regular problem sets **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IC. Students expected to precisely observe phenomena and accurately record findings by
- regularly justifying conclusions and claims in all written work **FA,IA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**
- engaging in daily collaboration to complete or check work **IA**

ID. Students expected to evaluate the soundness and relevance of information and reasoning findings by
- regularly justifying conclusions and claims in all written work **FA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**
- engaging in daily collaboration to complete or check work **IA**

IIA. Students identify unexamined cultural, historical and personal assumptions and misconceptions that impede and skew inquiry by
- identifying weaknesses or misconceptions in related prior mathematical concepts **IA**
- discussing problems from multiple perspectives and opposing views to determine validity to various approaches **IA**
- engaging in daily collaboration to complete or check work **IA**

IIIA. Students use appropriate technologies as extensions of the mind by
- exploring mathematical ideas and problem solving using tools such as graphing calculators, Winplot, Mathematica, Excel, etc. **IA**
- making mathematical conjectures based on reasoned exploration **IA**

IIIB. Students recognize, pursue, and explain substantive connections within and among areas of knowledge by
- applying calculus methods to familiar contexts, e.g., applying green's Theorem in physics **FA**
- solving problems that require similar means which involve new or less familiar application contexts and justifying conclusions **FA**

IVA. Students construct and support judgments based on evidence through
- experimenting with 3d graphs then generalizing structure **FA,IA**
- Hypothesizing and proving vector properties **FA**
- exploring and justifying solutions to problems in class on a daily basis **FA**

IVB. Students will be challenged to write and speak with economy, power, and elegance by
- supporting answers with written justification using precise mathematical notation and language **FA,IA**
- making sound mathematical verbal arguments using precise language **FA,IA**

IVC. Students will identify and characterize the composing elements of dynamic and organic wholes, structures and systems.
- Actively developing the theory of vector calculus **FA,IA**

IVD. Students will be challenged to develop an aesthetic awareness and capability.
- Looking at the historical development of vector calculus **IA**
- Comparing student solutions and discussing relative merits, including elegance **IA**

VA. Students will identify, understand and accept the rights and responsibilities of belonging to a diverse community by
- actively participating in class discussions **IA**
- respecting each others' questions and responses, both in and out of class **IA**
- collaborating outside of class on Take Home and other assignments without infringing on each others' intellectual capital **IA**

VB. In order for students to make reasoned decisions which reflect ethical standards, and act in accordance with those decisions, students
- collaborate outside of class on assignments without infringing on each others' intellectual capital **IA**
- produce their own work on formal assessments **FA**

Instructional Design and Approach:

Students should be involved in exploration of the concepts and topics through reading of the text and outside material, giving presentations to classmates, through computer work, and solving problems in individual and group settings. Students will be asked to engage in the dialogue of problem solving, and to help their classmates understand the content of the course. Learning to leverage both an algebraic and geometric understanding of the material is a major goal for the course.

Student Expectations:

Students are expected to come to each class prepared to contribute to the classroom learning experience. This involves reading all assigned text and attempting all assigned problems before coming to class. Students are expected to work together and independently in deepening their understanding of course concepts. Students will have to take careful and complete notes in class, as the text does not cover all required material.

Assessment Practices, Procedures, and Processes:

Students are assessed through a variety of means including, but not limited to: written in-class and out-of-class exams, quizzes, classroom presentations, and homework. The relative weights of these factors depend on the amount of data available from each type of assessment. Typically, the breakdown is 2/3 tests and quizzes, with 1/3 homework and other assignments. Semester grades are based on 80% of the term work grade and 20% from the final exam.

Sequence of Topics and Activities (approximate)

Weeks 1-3	Vectors
Weeks 4-6	Functions, graphs, limits, differentiability, partial and directional derivatives
Weeks 7-8	Differential geometry, vector fields
Weeks 9-10	Taylor series, differentials, optimization
Weeks 11-12	Iterated integrals
Weeks 13-15	Line integrals, Green’s Theorem, independence of path
Weeks 16-18	Surface integrals, Stokes theorem, Gauss’ Theorem

Following is the more detailed topic-by-topic syllabus given to students on the first day of class.

Multivariable Calculus

Micah Fogel

Most of the interesting things in the universe have more than one input or output. Very seldom do we get just one *x* producing just one *y*. Hence all the calculus we learned in the BC sequence isn't enough. In this course we will use all of the tools acquired in the BC calculus sequence to explore what happens when more variables are involved.

This course is NOT just a continuation of BC. You really need to know BC solidly to have a reasonable expectation of success in this course. MVC is rewarding, because you get to learn some really cool stuff. It is probably my favorite course to teach at IMSA. But it is also *hard*. You will have a lot of work, and even more unassigned studying if you want to make the most of this course.

If you need help, I'm easy to find. Just walk in the front door of the math office, and turn left. You can also call me at 907-5086, or e-mail me at fogel@imsa.edu.

Homework

There is a lot of it. You can expect one to two assignments per week that will be collected and graded. Homework accounts for approximately 1/3 of your grade each quarter. Additionally, each homework assignment has a reading assignment and recommended problems. The reading assignment should be considered required. In fact, you should try to read ahead if you can, and then re-read the indicated sections when the homework is handed out. (This will not always be possible since we do skip around in the text a little, but for the most part if we're working on one section, the following section will be up next.)

The recommended problems will not be graded, nor are they required. But they are highly recommended. You probably don't need to do all of them, but you should try an assortment. Many answers are in the back of the text and I have complete solution manuals to check your work and your understanding. Very few people can make it through this class successfully without going substantially beyond the minimum assigned work. Don't assume you're one of them until you get a whole bunch of A's!

Quizzes/Tests

There will be regular quizzes and tests (I don't really distinguish between the two). One every two weeks or so. They form the remaining 2/3 of your grade each quarter. There is also a final exam which will count 20% of your overall semester grade. If you are taking this course pass/fail and would be getting an A you do not have to take the final exam. Otherwise the final is mandatory.

Topics

Coordinates and Vectors—the geometry of many dimensions

3D space and *n*-space

- Cartesian coordinates and vectors
- Vector algebra and geometry; unit vectors

Inner product (or dot product)

- Geomteric definition and coordinate formula
- Algebraic properties: lengths, interaction with vector addition and scalar multiplication, Cauchy-Schwarz and triangle inequalities
- Projections and orthogonality
- Equations of lines and planes

Cross Product
- Geometric definition
- Algebraic properties
- Triple product: volume of parallelopipeds, determinant notation, determinant formula for cross product

Aside on matrix algebra

Multivariable functions

Graphs, sections, level curves

Limits and continuity

Differentiation

Partial derivatives
- Motivation, definition, and notation
- Chain rule, including restricted variation
- Derivative of $f(\mathbf{x} + t\mathbf{u})$; directional derivatives

The gradient
- Use in computing directional derivatives
- Notation
- Application

Differentiability
- Precise definition of differentiability, examples

Higher order derivatives and equivalence of mixed partials

The multivariable Taylor's series and min/max problems
- Taylor's formula and results on remainders
- Minima and maxima; critical points and their classification
    - Second derivative test
    - Constrained critical point theory; Lagrange multipliers

Vector valued functions and their derivatives
- Vector fields
- The derivative of a vector valued function
- When is a field a gradient?
- Geometry and algebra of divergence and curl

Integration

Multiple integrals
- Double integrals over a rectangle
    - Reduction to single integrals and Fubini's Theorem
- Double integrals over other regions
- Triple integrals over boxes
- Triple integrals over other regions

Alternate coordinate systems
- Change-of-variables formula

Integrals and vectors
- Path integrals
- Line integrals
- Fundamental theorem, part I
    - Green's Theorem
- Surfaces
    - Parameterized surfaces
    - The normal vector $d\mathbf{S}$
    - Surface area
    - Scalar functions on surfaces
    - Vector functions and flux

The Fundamental Theorem Part II
- Stokes' Theorem
- Conservative vector fields
- Gauss' Theorem