Comprehensive Course Syllabus

Mathematical Investigations II

Course Description:

The Mathematical Investigations courses integrate topics from all areas of pre-calculus mathematics. In these courses, students will be expected to explore mathematical concepts, make conjectures and present logical, valid arguments for mathematical assertions. Both written and oral forms of communication are emphasized. MI-2 is the second course in this sequence and will concentrate on the study of matrices, linear relationships and equations, functions, function transformations, exponential functions and combinatorics.

.

I N S T R U C T O R :

Matthew McCutcheon
mmccutcheon@imsa.edu
Section 1: 12:20 – 1:15 PM
Room: A156

Office Hours:
Often Modules 3 and 7. Also afterschool from 4:15-4:45 PM.

Students should take advantage of these times for individual consultation – the instructor enjoys doing this! Making an appointment would be helpful and would guarantee time for the student, however, this is not necessary.

Text(s) / Materials:
The Mathematical Investigations curriculum is a product of past and present IMSA mathematics faculty. Materials will be made available as needed.
Students are required to have a graphing calculator. TI-89 Titanium is recommended.
Students will also use *Mathematica, Desmos,* and various internet resources via their laptops.

Essential Content:

- Matrices as a way of organizing information
- Matrix operations as a means of interpreting contexts
- Definition of a linear relationship
- Recognition and interpretation of linear relationship from multiple representations
- Beginning of data analysis/regression strand
- Conceptual underpinnings of solving systems of equations
- Definition/concept of function, domain, range
- Function composition
- Multiple representations of functions
- Function transformations
- Inverse functions
- Exponential functions as a form of growth/decay
- Laws of exponents
- Basic counting principle
- Permutations and combinations

IMSA Math Standards addressed:

A. Students studying mathematics at IMSA demonstrate a disposition and propensity to use mathematics, a variety of problem solving strategies, and creative thought to solve problems by:

A.1	investigating and gaining insight into mathematical concepts by selecting and using a variety of traditional and creative problem solving strategies and methodologies. [SSL-I.B,III.B,III.C,IV.A]
A.2	posing, solving, and extending both multi-step routine and multi-step unconventional problems. [SSL-I.B,III.B,III.C,IV.A,IV.C]
A.3	interpreting, generalizing , and verifying the understanding gained in the problem solving process and extending it to new settings. [SSL-I.B,II.B,III.B,III.C,IV.A,IV.C]
A.4	using a variety of resources and problem solving approaches. [SSL-III.B,IV.A]
A.5	demonstrating confidence, persistence, and reflective analysis of the effectiveness of an approach when attempting to solve a problem. [SSL-I.D,II.A,II.B,III.B,IV.A,IV.C]

B. Students studying mathematics at IMSA reason logically in mathematical situations and understand the nature, role, and necessity of proof and counterexample in mathematical reasoning by:

B.2	reasoning inductively and deductively. [SSL-I.B,III.C,IV.A]
B.4	enhancing inductive and deductive reasoning through the use of intuition, imagination, and other forms of reasoning. [SSL-III.C,IV.A]

C. Students studying mathematics at IMSA communicate clearly and accurately about mathematical relationships and results by:

C.1 understanding mathematical information given in written, oral, symbolic, numeric, or graphic form and interpreting the relationship it represents. [SSL-IV.B]

C.2 accurately recording and effectively communicating using proper notation, vocabulary, and usage in a variety of modalities (written, oral, graphic, algebraic, etc.). [SSL-I.C,IV.B,V.A]

C.3 presenting mathematical work and results using the power of mathematical language effectively. [SSL-IV.B,V.A]

C.4 summarizing results in a form that is accurate, appropriate to the topic and level, and understandable to the intended audience. [SSL-I.C,IV.B,V.A]

D. Students studying mathematics at IMSA demonstrate awareness of the inter-connectedness of mathematical thought in inter- and intra-disciplinary settings by:

D.1 understanding that mathematics is a system of interconnected ideas. [SSL-III.B,III.C,IV.]

D.2 recognizing the commonalties among the components and processes of the sub-disciplines of mathematics. [SSL-I.B,III.B,III.C,IV.C]

E. Students studying mathematics at IMSA understand and employ the power, economy, clarity, and elegance of mathematical representations by:

E.1 recognizing that mathematical representations carry specific meanings and using mathematical notation correctly to enhance clarity and avoid ambiguity. [SSL-II.B,IV.B]

E.2 applying a variety of techniques to compare and manipulate mathematical representations. [SSL-I.A,III.B,IV.C]

E.3 recognizing the structure underlying a mathematical representation and utilizing this structure in analysis and problem solving. [SSL-III.B,IV.A,IV.C]

E.4 selecting an appropriate mathematical representation and demonstrating how it reflects the salient points of the situation it describes. [SSL-I.B,I.D,II.B,III.B,IV.A,IV.C]

F. Students studying mathematics at IMSA use and interpret appropriate mathematical models to represent real-world situations by:

F.1 choosing an appropriate representation or mathematical model for a given situation. [SSL-I.A,I.B,III.B,IV.A,IV.C; IL-AoL-5,7.C,8.A]

F.2 understanding and explaining the relationship between the model and the given situation. [SSL-I.B,III.A,III.B,IV.A,IV.B,IV.C]

F.3 analyzing and explaining how variations in the situation will affect the model and how parametric changes in the model would be reflected in the situation it describes. [SSL-I.B,I.D,III.A,III.B,IV.A,IV.B,IV.C]

F.4 interpreting mathematical results in terms of the situation modeled. [SSL-I.D,III.B,IV.A,IV.C]

G. Students studying mathematics at IMSA understand the underlying concepts and characteristics of mathematical functions and relations by:

G.1 demonstrating fundamental recognition and analysis of relations and functions and their characteristics. [SSL-I.A,IV.C; IL-8.B]

G.2 developing and using a toolbox of prototypical functions (linear, exponential). [SSL-I.A,III.B,IV.C; IL-8.B,9.D]

G.3 evaluating and manipulating functions, creating multiple representations for a single function. [SSL-I.A,IV.C; IL-8.A,8.B]

G.4 applying operations and transformations to functions and demonstrating how changes in one representation of a function affect other representations of that function. [SSL-I.A,III.B,IV.C]

G.5 modeling a given situation or data with an appropriate function, using the model to make predictions. [SSL-I.B,III.B,IV.A]

I. Students studying mathematics at IMSA understand and apply geometric relationships by:

I.3 selecting and using appropriate geometric relationships, properties, formulas, tools, and units when working in a geometric context. [SSL-II.A,III.B,IV.A,IV.C]

I.4 modeling situations geometrically in two and three dimensions to formulate, describe, and solve problems. [SSL-I.B,III.B,IV.A,IV.C]

I.5 performing and describing geometric transformations. [SSL-I.A,IV.B; IL-9.A]

I.6 solving problems involving coordinate (analytic) geometry. [SSL-I.A; IL-9.A,9.C]

J. Students studying mathematics at IMSA use data to research questions, inform decisions, and evaluate assertions by:

J.2 identifying, selecting, and using appropriate statistical and graphical tools to analyze data in a variety of contexts. [SSL-I.A,III.B,IV.A]

K. Students studying mathematics at IMSA understand and apply discrete mathematical models by:

K.1 using matrices, sequences, and their operations to model phenomena. [SSL-I.A,I.C,III.B]

K.2 analyzing and interpreting situations using recursive thinking and inductive reasoning. [SSL-I.A,I.B,IV.A]

K.3 creating and interpreting directed graphs and networks. [SSL-III.B,IV.C]

K.4 demonstrating an understanding of basic counting principles and the situations under which they

may be applied. [SSL-I.A,I.D,III.B]

L. Students studying mathematics at IMSA use technology to gain insight and obtain different perspectives on problems by:

L.1 deciding whether to use technology, selecting an appropriate technology for a given situation, and understanding the limitation of the technology. [SSL-I.D,II.A,III.A]

L.2 using technology to facilitate doing, exploring, and understanding of mathematics. [SSL-II.A,III.A,IV.A]

L.3 judging the reasonableness of information and answers given by technology. [SSL-III.A,IV.A]

SSLs and Outcomes:

FA: Formally assessed, IA: Informally assessed

IA. Students are expected to demonstrate automaticity in skills, concepts, and processes that enable complex thought by

- completing a basic algebra skills test with 100% accuracy **FA**
- completing weekly problem sets **FA**
- completing daily worksheets **IA**
- engaging in daily collaboration to complete work **IA**
- completing quizzes and tests **FA**

IB. Students are expected to construct questions, forge connections and deepen meaning by

- completing daily worksheets **IA**
- completing problem sets **IA**
- completing writing assignments **IA**
- creating original problems and their solutions **FA**
- conversing and collaborating with peers **IA**
- contributing to large group conversation **IA**
- solving problems that require a novel compilation of knowledge and skills on quizzes/tests **FA**

IC. Students are expected to precisely observe phenomena and accurately record findings

- through data-based/real-world explorations and projects **FA**
- by recognizing patterns within mathematical situations and problems

IIIA. Students use appropriate technologies as extensions of the mind by

- computing linear regressions and using this model to make predictions **FA**
- exploring mathematical ideas and problem solving using tools such as graphing calculators, Desmos, Mathematica, Excel, etc. **IA**
- producing a technology-generated product, including mathematical equations, graphs and text **FA**

IIIB. Students recognize, pursue, and explain substantive connections within and among areas of knowledge through

- regular participation in both small and large group discussions **IA**
- focused inquiries on weekly Problem Sets which require brief written responses **FA**
- extended expository writing assignments **FA**

IVA. Students construct and support judgments based on evidence by

- exploring mathematical relationships (e.g. definition of function, effects of various parameters in transforming functions via technology in search of patterns), from which they make conjectures, test their hypothesis, and justify mathematically **FA**
- completing linear regression analysis activities **FA**

IVB. Students will be challenged to write and speak with economy, power, and elegance through

- regular participation in both small and large group discussions **IA**
- focused inquiries on weekly Problem Sets which require brief written responses **FA**
- short answer responses to justify reasoning on quizzes and/or tests **FA**
- writing projects (usually paired activity) in which students explore, explain and formally present a problem and its solution or thorough explanation of an applied concept. **FA**

VA. Students will identify, understand and accept the rights and responsibilities of belonging to a diverse community by

- working collaboratively in groups on a daily basis to explore, discuss, and solve problems, utilizing each others' various strengths **IA**
- engaging in a formally assessed writing project(s) with a partner, which requires a team dynamic for successful completion **IA**

VB. In order for students to make reasoned decisions which reflect ethical standards, and act in accordance with those decisions, students

- learn to collaborate in class to learn and solve problems, but produce their own work for assessment **IA**
- collaborate outside of class on assignments in an appropriate manner, modeling their in-class behavior **IA**

Instructional Design and Approach:

The instructional design of the mathematics core course provides opportunities for students to work collaboratively on a regular basis both in and out of class. Collaboration encourages oral communication, multiple perspectives in problem solving, and self-regulation. Carefully crafted and sequenced questions, problems, and applications comprise our problem-centered curriculum, which enables learning through guided discovery. This process requires pattern recognition, mathematical reasoning and visualization, critical thinking, appropriate use of technology and use of multiple representations in building connections within and between mathematical concepts. Regular teacher feedback and ongoing assessment shapes the learning experience for each individual student. The teacher's informal assessment of each student and the class as a whole tailors instruction to immediate need, generates enthusiasm, and insures intended connections. In addition, students are expected to communicate their understandings in writing with clarity, coherence, and mathematical accuracy.

Alongside daily lessons, students are regularly working problem sets, which contain a mixture of review, practice, and novel problems from various mathematical topics. Students are expected to collaborate in working toward a solution, but final writing should be their own. Ultimately, students

are responsible for the material contained within the problem sets. Also integral to the core experience are unique classroom project(s) that are intermittently incorporated to introduce students to mathematical inquiry, stretching their understandings in new directions and possibly beyond core content. As a result of this carefully structured learning experience, students' abilities to engage in mathematical inquiry, pose questions, and communicate mathematical concepts evolve, inviting creativity in problem solving, application, and further collaboration.

Student Expectations:
All students are expected to

- be involved in class discussions and explorations.
- maintain a notebook containing class notes, homework assignments, Problem Sets, and other handouts.
- complete all assignments, problems sets and writing assignment(s) in a timely manner.
- take responsibility for learning certain basic skills and relationships.
- take responsibility for seeking additional help as it is needed

Materials needed:

1. Graphing calculator (TI-89 Titanium, TI-89, etc.)
2. Notebook (3-ring, at least 1.5 in.) with clearly defined section dividers for these components:
   a. classroom work and notes
   b. quizzes
   c. problem sets
3. Laptop, with Mathematica successfully installed

Bring the notebook and calculator with you to class everyday. The notebook is to be chronologically ordered and is to contain mathematics only.

Course components:

Daily worksheets and classroom work. This work must be completed on a timely basis, usually by the beginning of the next class period. Each student is responsible to contribute his or her fair share to the classroom learning experience. If a student comes to class unprepared, they jeopardize the integrity of the learning environment. That is, they do not merely cheat themselves; they cheat the other students at their table, and in fact, the entire class. We expect each student to have made an honest attempt at completing the class sheets handed out the previous class period. Students will be asked to engage in the dialogue of problem solving, and help their classmates understand the content of the course. The instructor will periodically check your notebooks. All problems should be worked and answers should be checked with your tablemates. Notebooks will be graded for completeness and correctness. When you miss class, it is your responsibility to find out what you missed and make up this work.

Communication: Talk to your instructor. The workload at IMSA may surprise you. There will be weeks were you are buried with work from all your classes. If you can't get a problem set done by the due date, talk with your instructor **before** the due date.

Problem Sets. If the problem sets are late the following penalties apply:

Not turned in by class time, but turned in by 4:30 p.m. of the same day	-20%
Not turned in on time, but turned in by noon the following class day	-50%
Turned in later than noon the following class day	-100%

Webwork. A short online portion of problem sets will usually be due each Tuesday and Friday at 5:00 pm. Late submissions will not be accepted, as the system will stop taking submissions by that time. This is counted as part of the Problem Set grade.

Projects. These may be varied in length, and are often designed to be done with a partner. Writing in mathematics is a focus, along with the mathematics involved. A project will replace the problem set for that given week(s).

Quizzes. These will usually cover 3-6 worksheets from within a unit and may be unannounced. This is part of individual assessments.

Exams. These will be announced well ahead of time and will usually be at the end of a unit. Some of the exams will consist of a calculator-use section, as well as a no-calculator-use section. You may not keep your exams, but may come into the Math Office to look them over any time. This is part of individual assessments.

Final Exam. This will be comprehensive, and will consist of a calculator-use section and a no-calculator-use section.

Assessment Practices, Procedures, and Processes:

Weights of components of the quarter grade:

Individual Assessments	60%
Weekly Problem Sets/Writing	25%
Notebooks, etc	15%

Weights of components of the semester grade:

Individual Assessments	48%
Weekly Problem Sets/Writing	20%
Notebooks, etc.	12%
Semester Final Exam	20%

Grading scale:

A - 90% or above
B - 80% or above but less than 90%
C - 70% or above but less than 80%
D - less than 70%

Note: Students who earn a C– or D will be required to take a Math Support Intersession during January 2018. Those students who earn an overall C average or earn a C or lower test average (including the final exam) will be evaluated case-by-case for possible requirement to take this intersession.

Sequence of Topics and Activities

Units 1-5 take 12-14 teaching days each, including assessments.

1. Matrices
2. Linear Thinking
3. Introductions to Functions
4. Function Transformations
5. Exponential Functions
6 .Combinatorics *(4-5 teaching days)*