Theory of Analysis
Spring 2020

Teacher: Micah Fogel
Meeting time: Mod 8
Meeting location: A156
Text: Basic Analysis: Introduction to Real Analysis, by Jiří Lebl. Available for download at `https://www.jirka.org/ra/`
Course Web Page: `staff.imsa.edu/~fogel/Analysis/`

Theory of Analysis is an extremely advanced course. You have all completed BC3 and most of you have completed at least one post-calculus elective. This course is going to look at the foundations of calculus—why it works and how all the pieces fit together. We will prove all the theorems that we talked about in calculus (IVT, EVT, MVT, FTC, etc.) but never really proved. We will justify that the tricks we use actually work (for example, can you really integrate a series term-by-term?). And we will find counterexamples and pathological twists that show where and why things break down.

You will be doing a lot of the work proving things. I will try on many occasions to provide a preview of the next day's presentation with some gaps that I will ask you to fill in. The next day I will ask various students to come to the board and present their solutions, proofs, and arguments.

In this class I will also ask you to write solutions to your homework using the LaTeX typesetting system. LaTeX is an incredibly powerful document preparation system that is designed to help you make your work look professional and be easily read by your intended audience. We will not learn the full power of this system (that would be a semester all by itself!) but will cover enough that you may decide to trade in MS Word or Googledocs for TeXShop or Overleaf.

Clearly we have a lot to learn, so let's get started!

Grades

The course grade will be broken down as follows:
Homework: 30%
In-class presentations: 30%
Quizzes and Tests: 40%
In addition there will be a final exam worth 20% of the overall course grade.

Topics Covered

Below is a rough listing of the topics I intend to cover. We will not make any attempt to stick to the schedule, so that if something is harder than expected we can take more time with it, and if something goes faster then we can forge ahead.

Week 1: Preliminaries
Intro to LaTeX, intro to proofs and writing. Notation we need to know.

Week 2: Meet the real numbers
Axioms for the real numbers. Properties of the real numbers. Basic theorems and relationships.

Week 3: Introduction to sets
Notation and operations. Functions and relations. Finite and infinite; countable and uncountable sets.

Weeks 4–5: Basics of point-set topology
Open and closed sets. Accumulation and adherent points. Metric spaces. Compactness.

Weeks 6–7 : Limits and Continuous Functions
Convergent sequences. Cauchy sequences. Limits of functions. Continuous functions. EVT. Connectedness and IVT. Uniform continuity.

Weeks 8–9: Differentiation
Definition and properties. Algebra and the chain rule. MVT.

Week 10: Taylor series
Taylor's formula with remainder.

Week 11-13: Integration
Riemann integrals. Basic properties. Integration by parts. Change-of-variables. Fundamental Theorem.

Week 14-15: Sequences of functions
Pointwise and uniform convergence. Differentiation and integration.

Week 16: Back to the real numbers
Constructing the real numbers from simpler objects.

Week 17: More advanced topics
Henstock/Kurzweil/Denjoy integral, consideration in multiple variables, complex numbers and analysis, Lebesgue integral, implicit function theorem, ...